Tracking studies of spin coherence in COSY in view of EDM polarization measurements

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abstract

- Measurements of the polarization of protons and deuterons in COSY have been analyzed to prepare for similar measurements for the Electric Dipole Moment (EDM) experiment being proposed at Brookhaven

- Spin tracking studies are presented, in particular the study of spin coherence time for polarization survival and possible methods to increase it
Strategy of measurements and of simulation

- In a vertical magnetic field, an injected longitudinally polarized proton, or deuteron, beam will show in time the appearance of a vertical polarization due to the EDM. The accuracy of polarization measurement (Onderwater talk, this conference) is

\[
\sigma_s \propto \frac{1}{PE\sqrt{NTA}}
\]

with \( P \), polarization, \( E \) electric field in particle rest frame, \( N \), number of particles, \( T \) time of measurement, \( A \) analyzing power of polarimeter.

- We want to make these quantities as large of possible, and in particular we need to keep coherence of spin oscillationis for a long time.

- Polarimetry studies in preparation for the EDM are performed on the low energy polarized proton and deuteron storage ring COSY at Jülich.

- Here we present simulation by spin tracking on a realistic model of the accelerator, to understand what are the limits of spin polarization survival in COSY and what are the causes and cures of spin decoherence.

- Following a proposal by Y. Orlov, one of us (F. Lin) has already addressed this latter issue in a preliminary study.
Polarized deuterons in COSY

Measurement in COSY were done on polarized deuterons injected with vector polarization “up” and stored at constant momentum and energy

\[ p_c = 0.97 \text{ GeV}, \quad G\gamma = -0.161003 \]

with \( G \) the magnetic anomaly (sometimes called \( a \))

Spin tracking is done in the simulation by two parallel processes:

1. Track the orbits of an herd of \( \text{rmacro} \) particles with first orbit transport matrices generated by MAD and higher order maps created by UAL-TEAPOT

2. Calculate the spin kick due to the field at each position, with SPINK matrices that describe the spin rotation in each machine element

- In the simulation the COSY lattice contains dipoles (with fringes), quadrupoles, sextupoles, and an RF cavity
Spin coherence and spin tune

Spin coherence simply means that spin oscillations should remain in phase for a large number of turns in the machine.

Spin decoherence produce an effective gradual depolarization of the beam during polarimeter measurements.

- Number of spin oscillations per turn is the spin tune $\nu_s$

- Coherence is achieved when the spin tune among all particles in a bunch remain approximatively the same. There are at least two different ways to calculate spin tune in simulation by spin tracking:

  1. Calculate the spectrum of spin oscillation frequency along a particle orbit by Fourier or Fast Fourier analysis or simply by counting spin oscillations

  2. Find spin tune from the one-turn spin matrix. This is the method we used in this work.
The code SPINK

- The tracking code SPINK is based on the relativistic Thomas-BMT equation for spin motion in an electromagnetic field

\[ \frac{dS}{dt} = S \times b \]  \hspace{1cm} (1)

that represents the rotation of the spin vector \( S \) around an axis \( b \). In turn, \( b \) is a function of the electromagnetic field at the instantaneous location of the particle.

- We transform Eq.(1) to a matrix equation, with \( M \) a \( 3 \times 3 \) matrix (for vector polarization)

\[ S = MS. \]  \hspace{1cm} (2)

- Each machine element gives a thin lens kick to the spin vector \( S \)
Spin Matrix

- Coefficients of the spin matrix are function of three angles: 
  \( \delta \mu \), spin angular kick 
  \( \theta \), latitude and \( \phi \), longitude, angles for the rotation axis

- SPINK uses the Frénet-Ferret coordinates along the synchrotron, with \( \hat{y} \) a vertical axis, \( \hat{x} \) radial and \( \hat{z} \) a longitudinal axis tangent to the trajectory of the synchronous particle

- In F-F the three angles are function of the canonical coordinates of the moving particle

\[
\mathbf{r} = (x, x', y, y', \Delta \phi = -c \Delta t, \Delta E/pc).
\] (3)

- Spin matrix elements are in turn a function of the space coordinates. The spin matrix has unitary determinant and represents exactly a rotation
One-turn spin matrix, OTM

- Multiply all the spin matrices in one machine turn to obtain the SPIN One-turn matrix (OTM), that represents the rotation in one turn of the \( \mathbf{S} \) vector in a 3-dimension space.

- The eigenvalues of the SPIN OTM give the spin tune \( \nu_s \) that represents the number of oscillations per turn of the spin vector:

\[
\nu_s = \frac{1}{2\pi} \cos \left( \frac{\text{Tr}(M_{OT}) - 1}{2} \right)
\]  

(4)

where \( \text{Tr}(M_{OT}) \) is the trace of the OTM.

- **Note:** Similarly, the 3 complex conjugate eigenvalues of the orbit OTM furnish the (fractional part) of the transverse betatron frequencies and of the longitudinal synchrotron frequency.
Behavior of the Spin Tune

- The spin tune so calculated oscillates, from turn to turn, around an average value, that will converge to some asymptotic value after many turns, and which we will use in the following as the "spin tune" $\nu_s$.

- The distribution of spin tune among the various macro particles in the simulation constitute the spin tune line of width

$$w_s = \sqrt{<\nu_s^2> - <\nu_s>^2}$$

- **Note:** Strictly speaking, because spin and orbital motion are coupled, the true "spin tune" should be defined as the value obtained from the eigenvalues of the SPIN OTM obtained in a "SPIN one-turn", i.e. when the particle re-assumes as close as possible the spatial phase space values that it had at the beginning ($r = r_0$). It can be shown by stroboscopic averaging that the this value coincides with the one defined above.
Example: Spin tune and its average

• Average spin tune calculated for 8 particles sitting on the contour of a phase space ellipse corresponding to an emittance of $6.4 \times 10^{-5}$ m.rad
Example: Oscillations of $S_x$ (radial spin)

- Oscillations of the radial component of spin $S_x$ oscillation in 10 turns of COSY
  $S_y$ remains close to 1 during all turns because we are far from any depolarizing resonance. COSY is 183.473 m long

- The spin executes slightly less than an integer number of oscillation per turn

- Note: $S_x$ make oscillations at all because the particle considered has been injected not exactly on the equilibrium orbit and hence it is not perfectly matched to the spin invariant axis at injection
Spin Decoherence

- Particles with different emittances and different momenta have a different spin tune, so the beam they represent has some degrees of decoherence.

- Spin oscillations becomes increasingly off-phase and the overall beam polarization decreases.

- Also, if the spin makes any transition due to spin resonances in the lattice or induced by the action of spin flipper devices, like RF dipoles or solenoid, transition or spin flipping do not happen simultaneously for all the particles.
**Emittance of single particles and spin tune**

- Example of asymptotic spin tune average value for a particle on ellipses of increasing emittance

- If the total beam emittance (normalized) encompasses phase space ellipses up to an r.m.s. value of $5.10^{-6}$ m.rad, $\nu_s$, the spread remains of the order of $2.10^{-6}$

![Graph showing single particles on ellipses with 30,000 turns, $\Delta p/p = 0, \Delta \phi = 0$, no longitudinal motion.](image-url)
Example: emittance of a bunch and spin tune

- A beam of 256 deuterons with r.m.s. emittance of $10^{-5}$ m.rad was created with random Gaussian distribution in the transverse space and no energy spread, and propagated for 30,000 turns.

- The figure shows the distribution of the resulting spin tune among particles. In 100,000 turns the oscillations of the spin may be off by $90^0$.

Spin tune vs. beam dimension $x$[m] and $x'$[rad]
Example of spin tune line

- r.m.s. emittance $= 10^{-7}$ m.rad: 256 sample particles, 30,000 turns

- $\Delta \nu_s \approx 1.7 \times 10^{-6}$ is the r.m.s value (spin tune linewidth), that expand the spin life limited only by emittance effects to about 300,000 turns or 0.3 sec

![Graph](image)

x-axis: relative spin tune for emittance of $10^{-7}$ m.rad

y-axis: AU
Momentum spread and spin coherence

- The momentum spread $\Delta E/pc$ of the beam has an effect on spin coherence because particles with different energies in the beam travel on paths of different lengths.

- Example: spin tune of a coasting deuteron beam with transverse emittance and energy spread.

Spin tune of one particle at $\epsilon = 10^{-7}$
$\Delta E/pc = 10^{-7}$ to $10^{-3}$
Coasting beam
Spin tune linewidth due to $\Delta E/pc$

- Coasting beam. Effect on spin coherence for an ensemble of 2048 deuterons of emittance $10^{-6}$ randomly extracted

- Left: Gaussian energy spread of rms $= \Delta E/pc = 10^{-4}$
  Right: Reduce energy spread to $\Delta E/pc = 10^{-6}$
Spin tune and synchrotron oscillations

- Activate the RF cavity: deuterons start performing synchrotron oscillations

- The figure shows the comparison of the evolution of spin tune vs. turn number without and with a RF cavity activated in COSY at a voltage of 30 KV and harmonic 1.

- The modulation of the spin tune by the RF is evident, as caused by the modulation of the particle energy

Upper: evolution of turn-by-turn spin tune vs. turn number w/o RF (black) and with RF (red)

Lower: spin tune differences
Spin tune linewidth with synchro oscillations

- Spin tune line

- Distribution of spin tune difference among 2048 particles. RF cavity on with 30 KV. $\Delta E/pc = 10^{-4}$. R.m.s. emittance $\epsilon = 10^{-6}$. Spin tune difference standard deviation $\sigma = 2.4389 \times 10^{-4}$. 
**Orbit amplitude, no energy spread**

- The longitudinal motion effect on spin tune is caused by the average different amplitude of the orbit due to synchrotron-betatron coupling.

- For comparison, see the amplitude of the orbit in the next two figures.

- Left: No long. motion. 512 particles, 30,000 turns. $\epsilon = 10^{-6}$. Upper curve: standard deviation [m] of radial motion. Lower curve: standard deviation of vertical motion.

- Right: Same case. Std.dev(x)$=0.00174123$.
Orbit amplitude, w/ energy spread

- When the longitudinal motion is activated, the pattern is as shown in the two figures

- Left: with longit. motion. 512 particles, 20,000 turns. $\epsilon = 10^{-6}$.
  Upper curve: amplitude [m] of radial motion.
  Lower curve: amplitude of vertical motion

- Right: Same case. Std.dev=0.00183167
Trajectory length vs. energy spread

\[ \Delta E/pc = 0 \quad \langle \Delta L \rangle \quad \text{std.dev. (L)} \]

\[
\begin{array}{ccc}
0 & 3.73877745E-07 & 0.00061455262 \\
1.10^{-4} & 3.79208632E-07 & 0.00061579901 \\
\end{array}
\]

The distribution of trajectory lengths among particles produces a distribution of spin tune among particles, and therefore decoherence of spin, which is markedly higher when the longitudinal motion of the particles is included.

Distribution of trajectory lengths among 1024 random particles with and with no longitudinal motion, 10,000 turns, \( \Delta E/pc = 0 \) and with a Gaussian energy distribution \( \Delta E/pc = 1.10^{-4} \) and RF on
Spin tune and bunch length

- Beam structure: $\Delta \phi = -c\Delta t \neq 0$. $\Delta \phi$ is the 5.th phase space coordinate. Another effect to be considered on spin decoherence is the beam bunch longitudinal structure.

- Bunch length effect on trajectory length, averaged over 128 particles. The effect is rather strong.

- Trajectory lengthening vs. beam bunch length. The beam density has a Gaussian transverse distribution and energy distribution and parabolic structure in longitudinal. Standard deviation over 128 sample random particles after 10,000 turns. Emittance $= 10^{-6}$, RF voltage 30 KV.

![Graph of trajectory length vs. bunch length](image)
Spin tune spread vs. bunch length

- Spin tune difference at turn end vs. beam bunch length. The beam density has a Gaussian transverse distribution and energy distribution and parabolic structure in longitudinal. Standard deviation over 128 sample random particles after 10,000 turns. R.m.s. emittance = 10^{-6}, RF voltage 30 KV
Correction of spin decoherence

In a realistic lattice and a realistic beam the spin tune is not the same for all particles, or, equivalently, there is always a degree of spin decoherence. Some of the causes just examined of the spin decoherence are

1. finite emittance of the beam
2. energy spread of the beam
3. finite bunch length

- They produce different orbits for the various particles, in particular they cause different length of the trajectories, so the number of oscillation of the spin per turn (spin tune) is not the same for all particles and the oscillations of the spin, assumed in phase at injection in the storage ring, are not in phase anymore after a certain number of revolutions.
Cures

- A first cure for item no. 1 above is to reduce the beam transverse emittance: increase the brightness of the beam. Start with a beam of low emittance at the source and avoid mechanisms of diffusion that can dilute the emittance.

- At low energy, space charge forces are important and produce an increase of the emittance. **Cooling** of the beam is an effective method to reduce the emittance, but it has limitations and problems.

- Spin decoherence may also be cured using multipolar lenses, say sextupoles, in the lattice, similar to what is done to correct chromaticity. Spin decoherence and chromaticity are **non linear phenomena** and can be corrected with non linear optical elements.
Analysis of spin decoherence causes

- Look at the structure of the vector function $b$ in the Thomas-BMT Eq.(1)

  In the absence of any electric field

  $$b = \frac{q}{\gamma m} [(1 + G \gamma) B_\perp + (1 + G) B_\parallel]$$

  (5)

  $q$ and $m$ are the charge and mass of the particle, $\gamma = E/mc^2$ is the Lorentz energy factor, $G$ the anomaly

- $B_\perp$ and $B_\parallel$ are the magnetic field components perpendicular and parallel to the velocity of the particle. The spin kick is proportional to the value of the local field, as well as it is functionally dependent on the energy of the particle.
Different quantities among particles in the beam

- (A) Because of finite emittance and energy spread of the beam, different particles perform different trajectories

Orbit of two particles in 3 turns, in a beam w/o energy spread (black), and with $\Delta E/pc = 10^{-4}$ (red). Lower curve is the lengthening of the trajectory, relative to the machine reference orbit length

- Because of (A), the (1) magnetic field as seen by the particles, the (2) length of the trajectory over which the spin tune is calculated, and the (3) particle energy are different among particles
Cures of spin decoherence due to orbit variations

- There is nothing obvious to do to cure spin decoherence due to different energies as they enter the coefficients of the BMT equation, except decrease the energy spread.

- Correction of the spin decoherence can be done trying to minimize the difference in orbit between particles of different energy and emittance. This is a nonlinear problem, since the Thomas-BMT coupled with the equations of motion gives rise to non linear oscillations.

- The minimization process is similar, although more complicated, to the problem of chromaticity correction in an accelerator, requiring the use of non linear machine elements.

- Yuri Orlov has studied this problem and proposed a solution for the pitch effect in the small storage ring proposed for the EDM, and some of us (mostly F. Lin) had performed simulation by spin tracking with UAL-SPINK.
Nonlinear cure of spin decoherence in COSY

A first pass

- Consider COSY and its sextupoles

- For spin tune correction the best position of the sextupoles in the lattice is where the beam transverse size is large (large values of the Twiss functions $\beta_x$ and $\beta_y$) and when the dispersion $\eta_x$ is large, because
  
  1. the sextupolar magnetic field increases with the square of the distance from the accelerator equilibrium orbit
  
  2. high dispersion produce large radial displacement of the off-energy particle orbit, thus making the sextupoles more effective
COSY lattice and its sextupoles

- COSY has 18 sextupoles mostly used for chromaticity correction. They are about in the right locations around the ring to serve also as spin decoherence correctors.

- The actual position of the sextupoles in the lattice is shown.

- 5 of them are located in correspondence of a maximum value of the dispersion, where a group of dipoles is, namely $S_{5}, S_{6}, S_{7}, S_{8}, S_{9}$.

- Similarly where a second group of dipoles is, we have $S_{14}, S_{15}, S_{16}, S_{17}, S_{18}$.

- Some other sextupoles, $S_{10}, S_{11}, S_{12}, S_{13}$ are in correspondence to high values of the beta function.
Example: COSY lattice

For this case: betatron tunes = 3.34211 and 3.83556

• Lattice of COSY and Twiss functions. The position of the 18 sextupoles is indicated. Dispersions is the green curve.
Spin coherence correction by 10 COSY sextupoles

- In a first set of simulations we used 10 sextupoles to try and decrease the spin tune line width

- Only using these in correspondence of the dispersion, with equal strength, for $\Delta E/p_c = 10^{-4}$, the spin tune line width showed a reduction of a factor of about 3 with respect to the linewidth with no sextupole. The best value of sextupole strength was

$$K_2 = -\frac{\partial^2 B/\partial r^2}{(B\rho)} = 0.05 \ [m^{-3}]$$

$B$, sextupoles' field, $(B\rho) = pc/q$, beam "rigidity"

- More systematic and extensive work is required to search for optimum values of all available sextupoles to further increase spin coherence
Spin tune linewidth with and w/o sextupoles

- Using COSY sextupoles. Upper curve: $<\nu_s>$ as a function of sextupole strength. Lower curve: spin tune linewidth. 
  $\epsilon = 10^{-6}(\text{r.m.s}), \Delta E/pc = 10^{-4}(\text{r.m.s})$
Spin coherence time

- In the examples above we have tracked particles for a variety of number of turns, to check spin tune averaging and collected the spin tune line value at the end of the run.

- Assume the spin coherence completely lost when the spin oscillation phase $\phi_s$ will slip by $\pi$ in r.m.s among the particles in the beam

$$\delta\phi_s = 2\pi\delta \nu_s N_d = \pi, \text{ or } N_d = 1/2\delta \nu_s$$

- Convert coherence time to coherence turn number using the length of COSY = 183.473 m and the speed of the deuterons $\beta = 0.459$, where the period of the machine is $1.33\mu$sec.

- Using the numbers of the last optimization, we can conclude that the spin coherence of a coasting beam with emittance $10^{-6}$ and energy spread $10^{-4}$ may remain coherent for

$$150 \cdot 10^6 \text{ turns, or } 205 \text{ sec}$$
Concluding remarks

- We expect other spin decoherence effects during extraction, effects related to errors in the lattice, noise of the power supplies, earth movements.

- We could arrive at a formulation where every effect is quantitatively expressed in a form like

  \[ \delta \nu_s = \delta \nu_s(\text{emittance}) + \delta \nu_s(\text{energyspread}) + \delta \nu_s(\text{noise}) + \ldots \]

  that will help to find a method to maximize coherence time.

- This will be addressed in further simulation studies.