TMDs in Drell-Yan processes

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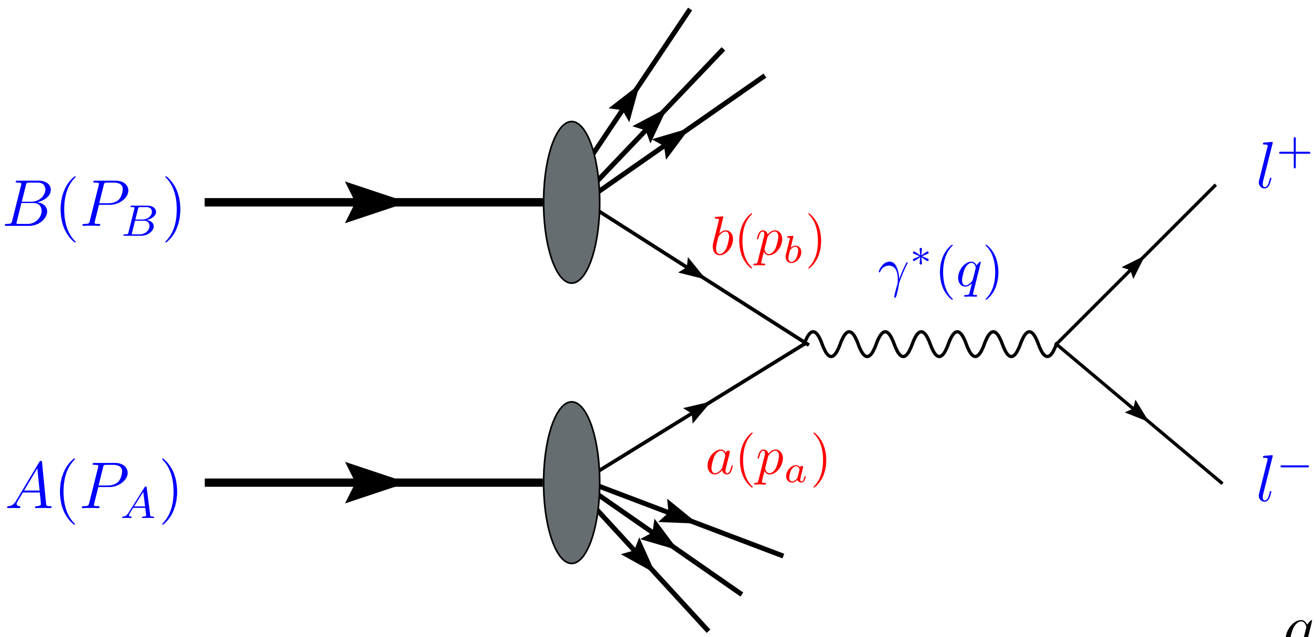
Collinear Kinematics

$$A + B \longrightarrow l^+ + l^- + X$$



$$P_A = \frac{\sqrt{s}}{2}(1, \mathbf{0}, 1)$$

$$P_B = \frac{\sqrt{s}}{2}(1, \mathbf{0}, -1)$$

$$p_a = \frac{\sqrt{s}}{2} x_a (1, \mathbf{0}, 1)$$

$$p_b = \frac{\sqrt{s}}{2} x_b (1, \mathbf{0}, -1)$$

$$q = p_a + p_b = (q_0, \mathbf{0}, q_L)$$

- $\hat{s} = (p_a + p_b)^2 = M^2 \equiv q^2$
- $y \equiv \frac{1}{2}\ln\left(\frac{q_0 + q_L}{q_0 - q_L}\right) = \frac{1}{2}\ln\left(\frac{x_a}{x_b}\right)$
- $\tau = x_a x_b = q^2/s$

Collinear Kinematics- Cross section

➢ Invariant cross section

$$\frac{d\sigma^{AB\to l^+l^-}}{d^4q} = \sum_{ab} \frac{1}{s}\, \hat{f}_{a/A}(x_a)\, \hat{f}_{b/B}(x_b)\, \hat{\sigma}^{ab\to l^+l^-}$$

- $x_a = \frac{q_0+q_L}{\sqrt{s}}; \qquad x_b = \frac{q_0-q_L}{\sqrt{s}};$
- $\hat{\sigma}^{q\bar{q}\to l^+l^-} = \frac{4\pi\alpha^2 e_q}{9q^2}$

➢ Angular distribution of a lepton in the photon (dilepton) rest frame

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^*} = \frac{3}{2(\lambda+3)}\,(1+\lambda\cos^2\theta^*)$$

- where $\lambda = 1$

Explored kinematical regions

$\Rightarrow \tau \equiv x_a x_b = q^2/s \equiv M^2/s$ gives the lower x that can be reached

- COMPASS($\pi^- p$):
  - π beam of $50 \div 200$ GeV/c, $s = 100 \div 400$ GeV^2
  - $M \simeq 4 \div 9$ GeV/c^2; for $s = 100$ GeV^2, $\tau = 0.16 \div 0.8$
    for $s = 400$ GeV^2, $\tau = 0.04 \div 0.2$
- RHIC(pp):
  - $s = 40000$ GeV^2
  - $M \simeq 4 \div 20$ GeV/c^2; $\tau = 4 \times 10^{-4} \div 1 \times 10^{-2}$
- PAX-GSI:($\bar{p}p$):
  - Collider: p beam of 3.5 GeV/c, $\bar{p}$ of 15 GeV/c, $s = 210$ GeV^2
  - $M \simeq 2 \div 9$ GeV/c^2, $\tau = 0.02 \div 0.4$
  - Fixed Target: $\bar{p}$ beam of 15 (or more) GeV/c, $s = 30$ GeV^2
  - $M \simeq 2 \div 4$ GeV/c^2, $\tau = 0.13 \div 0.53$

Transverse motion I

➢ If we consider the transverse motion of partons then:

$$p_a = \frac{\sqrt{s}}{2} x_a \left(1 + \frac{k_{\perp a}^2}{x_a^2 s}, \frac{2\boldsymbol{k}_{\perp a}}{x_a \sqrt{s}}, 1 + \frac{k_{\perp a}^2}{x_a^2 s}\right)$$

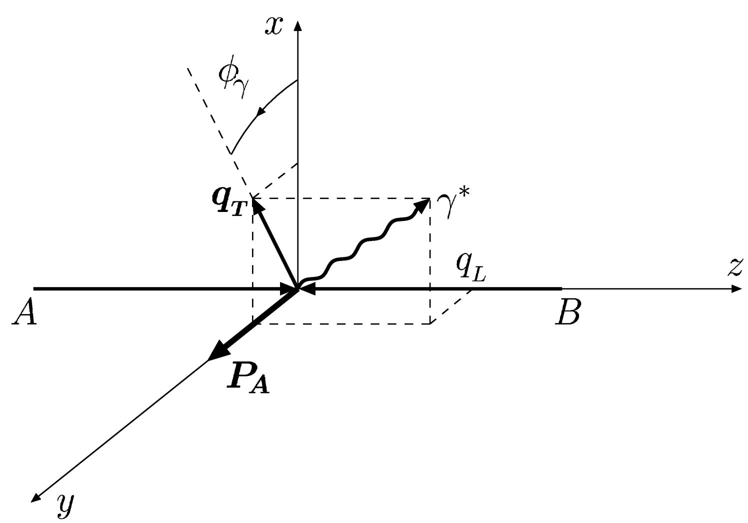
$$p_b = \frac{\sqrt{s}}{2} x_b \left(1 - \frac{k_{\perp b}^2}{x_b^2 s}, \frac{2\boldsymbol{k}_{\perp b}}{x_b \sqrt{s}}, -1 + \frac{k_{\perp b}^2}{x_b^2 s}\right)$$

➢ ... and the γ^* (dilepton) momentum has a transverse component in the h.c.m. frame

$$q = p_a + p_b = (q_0, \boldsymbol{q}_T, q_L)$$

✎ $\boldsymbol{q}_T = \boldsymbol{k}_{\perp a} + \boldsymbol{k}_{\perp b}$

Only low q_T ($q_T^2 \ll q^2$) have a non-perturbative origins



Transverse motion II

➢ Collinear Parton Distribution Functions:

$$f_{a/A}(x_a),\, \Delta q(x_a),\, h_1(x_a)$$

➢ Transverse Momentum Dependent (TMD) PDF:

$$A \text{ unpolarized: } \hat{f}_{a/A}(x_a, k_{\perp a}),\, \underbrace{\Delta \hat{f}^{a}_{s_y/A}(x_a, \boldsymbol{k}_{\perp a})}_{\text{Boer-Mulders}}$$

$$A^{\uparrow}\text{: } \underbrace{\Delta \hat{f}_{a/A^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})}_{\text{Sivers}},\, \underbrace{\Delta \hat{f}^{q}_{s_x/\uparrow}(x_a, \boldsymbol{k}_{\perp a}),\, \Delta^{-} \hat{f}^{a}_{s_y/\uparrow}(x_a, \boldsymbol{k}_{\perp a})}_{h_1}$$

$$\vec{A}\text{: } \underbrace{\Delta \hat{f}^{q}_{s_z/+}(x_a, \boldsymbol{k}_{\perp a})}_{\Delta q},\, \Delta \hat{f}^{q}_{s_x/+}(x_a, \boldsymbol{k}_{\perp a})$$

Unpolarized DY cross section

➢ New terms in the unpolarized cross section:

$$d\sigma \propto \Big[\tfrac{1}{2}\,\hat{f}_{a/A}(x_a,k_{\perp a})\,\hat{f}_{b/B}(x_b,k_{\perp b})\,d\hat{\sigma}^U + 2\,\underbrace{\Delta\hat{f}^{a}_{s_y/A}(x_a,\boldsymbol{k}_{\perp a})\Delta\hat{f}^{b}_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})}_{\text{Boer-Mulders}}\,d\hat{\sigma}^{T1}\Big]$$

where:

$$d\hat{\sigma}^U = |M^0{}_{-+;-+}|^2 + |M^0{}_{-+;+-}|^2$$

$$d\hat{\sigma}^{T1} = M^0{}_{-+;-+}M^0{}_{-+;+-}\cos\xi$$

Invariant unpolarized DY cross section

➢ Let us give a parametrization to the PDF's:

✎ $$\hat{f}_{a/A}(x_a, k_{\perp a}) = f_{a/A}(x_a)\frac{e^{-\frac{k^2_{\perp a}}{\langle k^2_{\perp a}\rangle}}}{\pi\langle k^2_{\perp a}\rangle}$$

✎ $$\Delta\hat{f}_{a^\uparrow/A}(x_a, k_{\perp a}) = (\sqrt{2e})\Delta f_{a^\uparrow/A}(x_a)\frac{k_{\perp a}}{M_{BM}}\frac{e^{-\frac{k^2_{\perp a}}{\langle k^2_{\perp a}\rangle_{BM}}}}{\pi\langle k^2_{\perp a}\rangle}$$

where:

$$\frac{1}{\langle k^2_{\perp a}\rangle_{BM}} = \frac{1}{\langle k^2_{\perp a}\rangle} + \frac{1}{M^2_{BM}}$$

➢ ...and try an analytical approximated integration:

✎ $$\frac{d^4\sigma}{d^4q} \simeq f_{a/A}\left(\frac{q_0+q_L}{\sqrt{s}}\right) f_{b/B}\left(\frac{q_0-q_L}{\sqrt{s}}\right)\frac{1}{2\pi\langle k^2_{\perp a}\rangle}\frac{4\pi\alpha^2 e_q^2}{9q^2 s}\left[1+O\left(\frac{q_T^2}{q^2}\right)\right] e^{-\frac{q_T^2}{2\langle k^2_{\perp}\rangle}}$$

Angular distribution of the unpolarized DY

➢ C.S. Lam and W. Tung, Phys. Rev. D 18,2447:

$$\frac{d\sigma}{d^4q d\Omega^*} = \frac{1}{32\pi^4 q^2 s^2}\Big[W_T(1+\cos^2\theta^*) + W_L(1-\cos^2\theta^*) + W_\Delta \sin 2\theta^* \cos\phi^* + W_{\Delta\Delta}\sin^2\theta^* \cos\phi^*\Big]$$

➢ J.C. Collins and D.E. Soper, Phys. Rev. D 16,2219

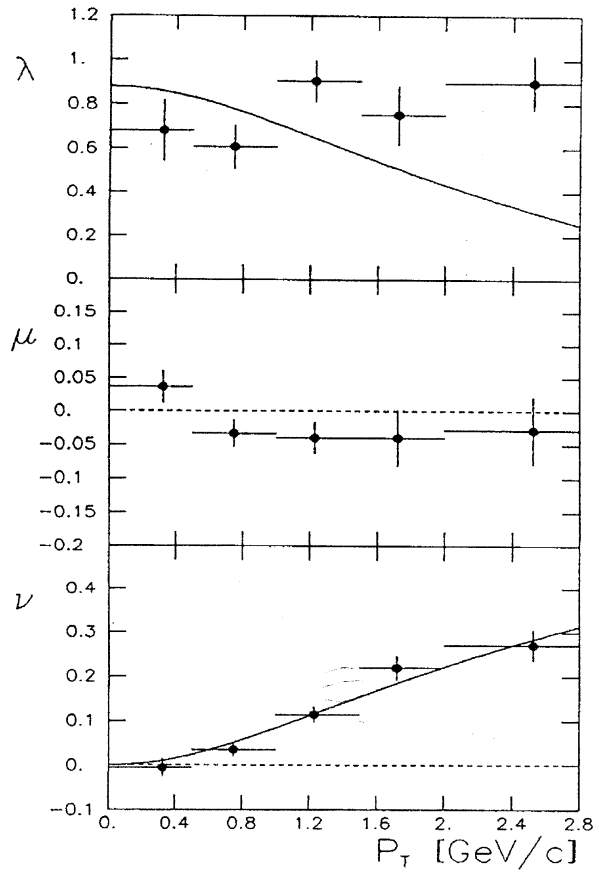
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^*} = \frac{3}{4\pi(\lambda+3)}\Big[1+\lambda\cos^2\theta^* + \mu\sin 2\theta^*\cos\phi^* + (\nu/2)\sin^2\theta^*\cos\phi^*\Big]$$

➢ Lam-Tung sum rule:

$$1-\lambda = 2\nu$$

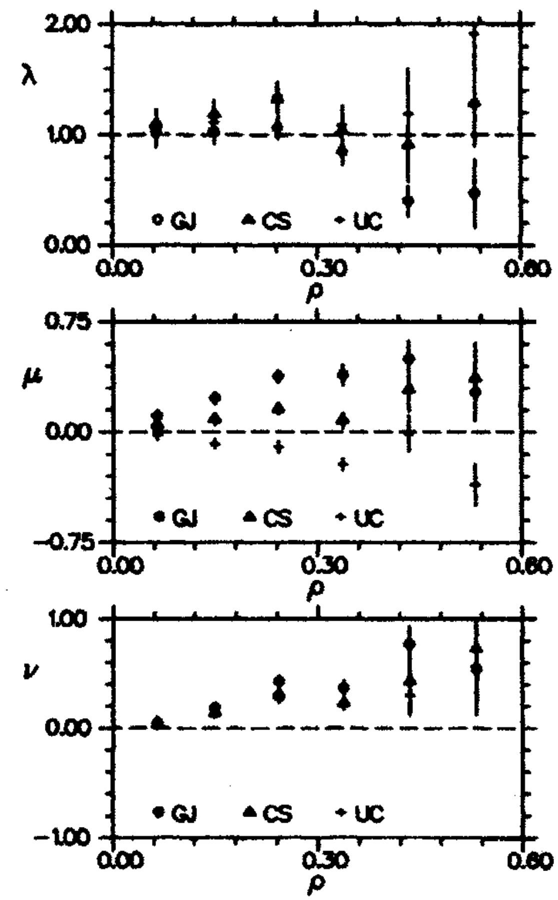
Angular distribution of the unpolarized DY: Experiments

➢ NA10: $\pi^-(194\text{GeV}/c)W \to \mu^+\mu^-$



Collins Soper frame

➢ E615: $\pi^-(252\text{GeV}/c)W \to \mu^+\mu^-$



$\rho = q_T/M$

Angular distribution of the unpolarized DY

➢ $\nu \neq 0$, $\lambda \neq 1$, $(1-\lambda-2\nu) \neq 0$:

✎ TMD: Boer-Mulders Effect

$$d\sigma \propto \Big[\tfrac{1}{2}\,\hat{f}_{a/A}(x_a, k_{\perp a})\,\hat{f}_{b/B}(x_b, k_{\perp b})\,d\hat{\sigma}^U + 2\,\underbrace{\Delta\hat{f}^{a}_{s_y/A}(x_a, \boldsymbol{k}_{\perp a})\Delta\hat{f}^{b}_{s_y/B}(x_b, \boldsymbol{k}_{\perp b})d\hat{\sigma}^T}_{\text{Boer-Mulders Effect}}\Big]$$

➢ Again, we give a parametrization to the PDF's and under the approximation $q_T^2 \ll q^2, q_L^2, q_0^2$ we try to find an analytical expression of the angular distribution in the dilepton helicity rest frame

Angular distribution of the unpolarized DY

➢ $$\frac{d\sigma^{f_{a/A} f_{b/B}}}{d^4 q d\Omega^*} = \frac{1}{2s} f_{a/A}\left(\frac{q_0+q_L}{\sqrt{s}}\right) f_{b/B}\left(\frac{q_0-q_L}{\sqrt{s}}\right) \frac{e_q^2 \alpha^2 e^{-\frac{q_T^2}{2\langle k_\perp^2\rangle}}}{12\langle k_\perp^2\rangle \pi q^2} \times \Big\{ \underbrace{1+\cos^2\theta^*}_{\lambda \text{ like term}} - \underbrace{\frac{q_T}{q}\left[\frac{q^2+q^{+2}}{q^{+2}-q^2}\right]\sin(2\theta^*)\cos(\phi^*)}_{\mu \text{ like term}} \Big\} \quad (1)$$

➢ $$\frac{d\sigma^{BM\ BM}}{d^4 q d\Omega^*} = -\frac{2e}{s}\Delta f_{a^\uparrow/A}\left(\frac{q_0+q_L}{\sqrt{s}}\right)\Delta f_{b^\uparrow/B}\left(\frac{q_0-q_L}{\sqrt{s}}\right) \times \frac{\langle k_\perp^2\rangle_{BM} e_q^2 \alpha^2 e^{-\frac{q_T^2}{2\langle k_\perp^2\rangle_{BM}}}}{96\pi\langle k_\perp^2\rangle^2 q^2} \times \frac{q_T^2}{M_{BM}^2}\Big\{ \underbrace{\sin^2\theta^*\cos(2\phi^*)}_{\nu \text{ like term}} + \underbrace{\frac{q_T}{q}\left[\frac{q^2+q^{+2}}{q^{+2}-q^2}\right]\sin(2\theta^*)\cos\phi^*}_{\mu \text{ like term}} \Big\} \quad (2)$$

where $q^+ = q_0 + q_L$

Single transverse DY cross section

➢ Single transverse spin asymmetry:

$$A_N = \frac{d\sigma^{A^\uparrow B \to l^+ l^-} - d\sigma^{A^\downarrow B \to l^+ l^-}}{d\sigma^{A^\uparrow B \to l^+ l^-} + d\sigma^{A^\downarrow B \to l^+ l^-}} = \frac{\Delta d\sigma^{A^\uparrow B \to l^+ l^-}}{2 d\sigma^{AB \to l^+ l^-}}$$

➢ In the collinear approximation $A_N = 0$

Single transverse DY cross section

➢ Numerator of the single transverse spin asymmetry:

$$\Delta d\sigma^{A^\uparrow B\to l^+l^-} \propto \Big[\frac{1}{2}\underbrace{\Delta\hat{f}_{a/A^\uparrow}(x_a,\boldsymbol{k}_{\perp a})}_{\text{Sivers}}\,\hat{f}_{b/B}(x_b,k_{\perp b})\,d\hat{\sigma}^U$$

$$+\,2\,\underbrace{\Delta^-\hat{f}^a_{s_y/\uparrow}(x_a,\boldsymbol{k}_{\perp a})}_{\text{“transversity”}}\,\underbrace{\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})}_{\text{Boer-Mulders}}\,d\hat{\sigma}^{T1}$$

$$+\,2\,\underbrace{\Delta\hat{f}^a_{s_x/\uparrow}(x_a,\boldsymbol{k}_{\perp a})}_{\text{“transversity”}}\,\underbrace{\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})}_{\text{Boer-Mulders}}\,d\hat{\sigma}^{T2}\Big]$$

✎ $h_1(x_a,\boldsymbol{k}_{\perp a}) = \Delta^-\hat{f}^a_{s_y/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\cos\phi_a + \Delta\hat{f}^a_{s_x/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\sin\phi_a$

and where:

$$d\hat{\sigma}^U = |M^0_{-+;-+}|^2 + |M^0_{-+;+-}|^2$$

$$d\hat{\sigma}^{T1} = M^0_{-+;-+}M^0_{-+;+-}\cos\xi$$

$$d\hat{\sigma}^{T2} = M^0_{-+;-+}M^0_{-+;+-}\sin\xi$$

Numerator of the transverse single spin asymmetry

➢ Let us give a parametrization to the PDF's:

$$\Delta^N \hat{f}_{a/A^\uparrow}(x_a, \boldsymbol{k}_{\perp a}) = (\sqrt{2e})\Delta^N f_{a/A^\uparrow}(x_a)\frac{k_{xa}}{M_{Siv}}\frac{e^{-\frac{k^2_{\perp a}}{\langle k^2_{\perp a}\rangle_{Siv}}}}{\pi\langle k^2_{\perp a}\rangle}$$

$$h_1(x_a, k_{\perp a}) = h_1(x_a)\frac{e^{-\frac{k^2_{\perp a}}{\langle k^2_{\perp a}\rangle}}}{\pi\langle k^2_{\perp a}\rangle}$$

where:

$$\frac{1}{\langle k^2_{\perp a}\rangle_{Siv}} = \frac{1}{\langle k^2_{\perp a}\rangle} + \frac{1}{M^2_{Siv}}$$

➢ ...and try an analytical approximated integration:

$$\frac{\Delta d\sigma}{d^4q} \simeq \Delta^N f_{a/A^\uparrow}\left(\frac{q_0+q_L}{\sqrt{s}}\right) f_{b/B}\left(\frac{q_0-q_L}{\sqrt{s}}\right)\, e^{-\frac{q_T^2}{\langle k^2_\perp\rangle + \langle k^2_\perp\rangle_{Siv}}}$$

$$\times \frac{q_T}{M_{Siv}}\frac{4\langle k^2_\perp\rangle_{Siv}\,\alpha^2 e_q^2}{9s\langle k^2_\perp\rangle(\langle k^2_\perp\rangle + \langle k^2_\perp\rangle_{Siv})^2 q^2}\cos\phi_\gamma\left[1 + O\left(\frac{q_T^2}{q^2}\right)\right]$$

Transverse DY: Angular distribution

- $$\frac{\Delta d\sigma^{\text{Sivers}}}{d^4q d\Omega^*} \propto \underbrace{\Delta^N f_{a/A^\uparrow}\left(\frac{q_0+q_L}{\sqrt{s}}\right)}_{\text{Sivers}} f_{b/B}\left(\frac{q_0-q_L}{\sqrt{s}}\right)$$
$$\times \frac{q_T}{M_{Siv}}\left\{ \underbrace{(1+\cos^2\theta^*)\cos\phi_\gamma}_{\text{analogous of the } \lambda \text{ like term in unp. DY}} + O(q_T/q)\right\} \quad (3)$$

- $$\frac{\Delta d\sigma^{h_1-BM}}{d^4q d\Omega^*} \propto \underbrace{h_1\left(\frac{q_0+q_L}{\sqrt{s}}\right)\Delta f_{b^\uparrow/B}\left(\frac{q_0-q_L}{\sqrt{s}}\right)}_{\text{transversity} \times \text{Boer-Mulders}}$$
$$\times \frac{q_T}{M_{BM}}\left\{\underbrace{\sin^2\theta^*\cos(\phi_\gamma+2\phi^*)}_{\text{anal. } \nu \text{ like term}} + O(q_T/q)\right\} \quad (4)$$

Double transverse DY cross section

- Double transverse spin asymmetry:

$$A_{TT} = \frac{d\sigma^{A^\uparrow B^\uparrow \to l^+ l^-} - d\sigma^{A^\uparrow B^\downarrow \to l^+ l^-}}{d\sigma^{A^\uparrow B^\uparrow \to l^+ l^-} + d\sigma^{A^\uparrow B^\downarrow \to l^+ l^-}} = \frac{\Delta d\sigma^{A^\uparrow B^\uparrow \to l^+ l^-}}{2 d\sigma^{AB \to l^+ l^-}}$$

- In the collinear approximation there is one contribution...

Double transverse DY cross section

$$
\begin{aligned}
\Delta d\sigma^{A^\uparrow B^\uparrow \to l^+ l^-} \propto \Bigg\{ & \frac{1}{2} \hat{f}_{a/A}(x_a, k_{\perp a}) \underbrace{\Delta^N \hat{f}_{b/B^\uparrow}(x_b, \boldsymbol{k}_{\perp b})}_{\text{Sivers}} \, d\hat{\sigma}^U \\
+ & \frac{1}{4} \underbrace{\Delta^N \hat{f}_{a/A^\uparrow}(x_a, \boldsymbol{k}_{\perp a})}_{\text{Sivers}} \underbrace{\Delta^N \hat{f}_{b/B^\uparrow}(x_b, \boldsymbol{k}_{\perp b})}_{\text{Sivers}} \, d\hat{\sigma}^U \\
- & \underbrace{\Delta \hat{f}^a_{s_z/\uparrow}(x_a, \boldsymbol{k}_{\perp a})}_{g_{1T}^\perp} \underbrace{\Delta \hat{f}^b_{s_z/\uparrow}(x_b, \boldsymbol{k}_{\perp b})}_{g_{1T}^\perp} \, d\hat{\sigma}^U \\
+ & 2 \Delta \hat{f}^a_{s_y/A}(x_a, k_{\perp a}) \, \Delta^- \hat{f}^b_{s_y/\uparrow}(x_b, \boldsymbol{k}_{\perp b}) \, d\hat{\sigma}^{T1} \\
+ & 2 \Delta \hat{f}^a_{s_y/A}(x_a, k_{\perp a}) \, \Delta \hat{f}^b_{s_x/\uparrow}(x_b, \boldsymbol{k}_{\perp b}) \, d\hat{\sigma}^{T2} \\
+ & 2 \, \Delta^- \hat{f}^a_{s_y/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \, \Delta^- \hat{f}^b_{s_y/\uparrow}(x_b, \boldsymbol{k}_{\perp b}) \, d\hat{\sigma}^{T1} \\
+ & 2 \, \Delta \hat{f}^a_{s_x/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \, \Delta \hat{f}^b_{s_x/\uparrow}(x_b, \boldsymbol{k}_{\perp b}) \, d\hat{\sigma}^{T1} \\
+ & 2 \, \Delta^- \hat{f}^a_{s_y/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \, \Delta \hat{f}^b_{s_x/\uparrow}(x_b, \boldsymbol{k}_{\perp b}) \, d\hat{\sigma}^{T2} \\
- & 2 \, \Delta \hat{f}^a_{s_x/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \, \Delta^- \hat{f}^b_{s_y/\uparrow}(x_b, \boldsymbol{k}_{\perp b}) \, d\hat{\sigma}^{T2} \Bigg\}
\end{aligned}
\quad (5)
$$

Terms 4–5 → ”Boer-Mulders“ × ”transversity”

Terms 6–9 → ”transversity“ × ”transversity”

Conclusions

➢ The DY process is a unique chance to study:

- ◇ the intrinsic partonic transverse momentum (unpolarized cross section);
- ◇ the Boer-Mulders function (unpolarized angular distribution);
- ◇ the Sivers function and its sign (transverse polarized DY)
- ◇ the transversity function, if some pieces of information on the BM function are known (transverse polarized angular distribution);
- ◇ the transversity function (double polarized DY)

➢ In order to perform this kind of study:

- the kinematical region should be similar to that of COMPASS or PAX
- the initial hadrons should be proton and anti-proton

Unpolarized cross section

$$\begin{aligned}
W_T^i &= \frac{1}{2}(1+\cos^2\beta_{ij})W_T^j + \frac{1}{2}\sin^2\beta_{ij}W_L^j - \frac{1}{2}\sin(2\beta_{ij})W_\Delta^j + \frac{1}{2}\sin^2\beta_{ij}W_{\Delta\Delta}^j \\
W_L^i &= \sin^2\beta_{ij}W_T^j + \cos^2\beta_{ij}W_L^j + \sin(2\beta_{ij})W_\Delta^j - \sin^2\beta_{ij}W_{\Delta\Delta}^j \qquad (6) \\
W_\Delta^i &= \frac{1}{2}\sin(2\beta_{ij})W_T^j - \frac{1}{2}\sin(2\beta_{ij})W_L^j + \cos(2\beta_{ij})W_\Delta^j - \frac{1}{2}\sin(2\beta_{ij})W_{\Delta\Delta}^j \\
W_{\Delta\Delta}^i &= \frac{1}{2}\sin^2\beta_{ij}W_T^j - \frac{1}{2}\sin^2\beta_{ij}W_L^j + \frac{1}{2}\sin(2\beta_{ij})W_\Delta^j + \frac{1}{2}(1+\cos^2\beta_{ij})W_{\Delta\Delta}^j
\end{aligned}$$